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Determining Representative Stress and Representative Strain in Deriving Indentation Flow Curves Based on Finite Element Analysis

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Abstract. A new method [1] to evaluate indentation flow curves using an instrumented indentation test has been applied to many materials for several years. Though the method produces relatively good results compared to uniaxial tensile tests, a few parameters had not been verified by theoretical or numerical methods. In this study, proportional constants of representative strain and representative stress were verified using finite element analysis and proven to be unaffected by the elastic property and strain level. The constants were generally dependent on the plastic property; however, one combination of the constants is independent of all properties. The values of this combination are consistent with early research and produced overlapping indentation flow curves with uniaxial curves.

Introduction

Interest in instrumented indentation test has increased rapidly over the past several decades. As an advanced technique originating from conventional hardness tests, the instrumented indentation test retains all the merits of the conventional hardness test: simple method, short test time, easy sample preparation, nondestructive test, almost no limitation of specimen size. The load-depth curve in Fig. 1 is general raw data from the test, from which various mechanical properties can be obtained.

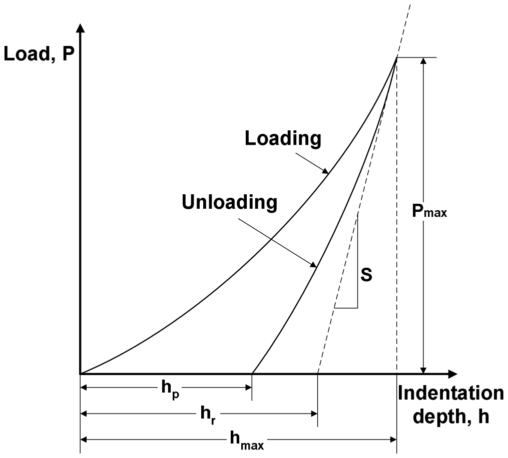


Fig. 1 A schematic diagram of a load-depth curve

By dint of much research, the instrumented indentation test is being standardized [2]. However, this standard deals only with hardness and elastic modulus, which are insufficient for evaluating mechanical properties. Generally, tensile properties are considered the most important mechanical properties. The general method for evaluating them is the uniaxial tensile test, which is destructive and difficult to apply to small materials and structures in-use. To overcome these demerits, a new instrumented indentation method was suggested that uses a spherical indenter [1]. It is very important to define representative stress and representative strain in this method. The authors used the concept that representative stress and representative strain are linearly proportional to mean contact pressure and maximum shear strain, respectively. Though these proportional constants play a significant role, they were not verified by a physical concept. Therefore, the optimum values must be determined and verified on the basis of a theoretical approach.

Theoretical background

The general algorithm for deriving the indentation flow curve is relatively clear and simple, as shown in Fig. 2. First, the load-depth curve is measured and the contact depth is obtained by calibrating elastic and plastic deformation around the indenter. The representative stress and representative strain are calculated and fitted to a certain constitutive equation. Finally, the indentation flow curve is obtained. The details of calculating the representative stress and representative strain are given below.

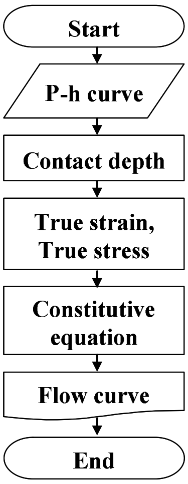


Fig. 2 Flow chart: evaluating the indentation flow curve.

The strain distribution can be obtained by analyzing the deformed shape beneath the indenter. The shear strain is derived by differentiating the displacement in the depth direction. The maximum shear strain is obtained at the contact point between the material surface and the indenter. By multiplying by a constant α, we can obtain the representative strain:

$$\varepsilon = \frac{\alpha}{\sqrt{1-(a_c/R)^2}}\frac{a_c}{R}, \tag{1}$$

where a_c is the contact radius and R the indenter radius. This equation seems wrong because when $a_c/R = 1$, $\varepsilon = \infty$. However, the ratio of indentation depth and indenter radius (h/R) is generally set in the range 0.2 to 0.6 for the ball indentation [3].

Tabor showed that the representative stress is proportional to the mean contact pressure in the fully plastic regime [4]:

$$\sigma = \frac{1}{\Psi}\frac{P}{\pi a_c^2}. \tag{2}$$

Early research [1] suggested values for α and ψ (plastic constraint factor) of 0.10 and 3.0, respectively. However, these values were determined when the indentation flow curves overlapped with uniaxial tensile curves. They thus have no physical meaning and produces unreliable indentation flow curves in a few cases. Therefore, in this study we investigated what properties affect the constants and determined their optimum values using finite element analysis.

Finite Element Analysis

The commercial code ABAQUS was adopted for finite element analysis of the instrumented indentation test. The specimen was assumed to be 100mm×100mm and constrained as shown in Fig. 3. The mesh was designed using four-node 3738 elements and a 2D axisymmetric model was used for fast calculation. A fine mesh was created to describe extreme deformation near the indenter. The indenter radius was 0.5 mm and assumed perfectly rigid. Thirty-eight materials, including 14 real materials and 24 ideal materials, were simulated. All materials followed power-law hardening behavior and have yield strength range of 200MPa to 800MPa, elastic modulus of 100GPa to 400GPa, and work-hardening exponent of 0.05 to 0.5; this covers almost all metallic materials.

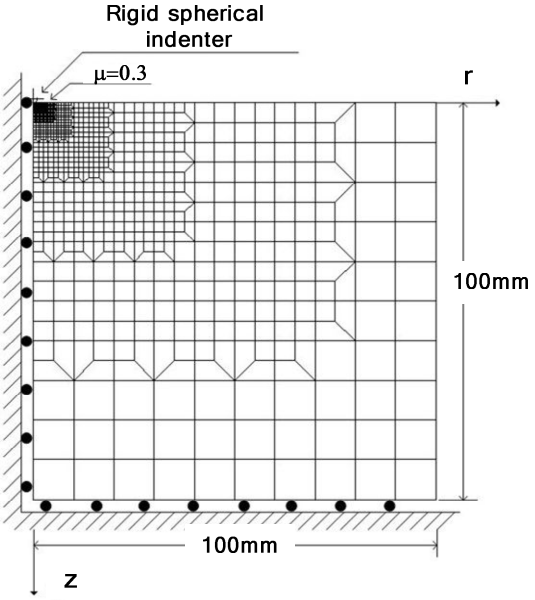


Fig. 3 Mesh and boundary conditions for finite element analysis

Result and Discussion

Since the proportional constants, α and ψ, can affect each other, it is almost impossible to determine optimum values separately. Based on the definition of plastic constraint factor, the values of ψ were calculated for various values of α in all simulated materials as shown in Fig. 4. The exact contact radius can be obtained by observing deformation morphologies in the ABAQUS postprocessor.

We considered three kinds of parameters: the elastic property, strain level and plastic property. First, the elastic property can be expressed the ratio of yield strength to elastic modulus, or the yield strain. Second, the strain level or effect of the indentation depth (which is a merit and demerit of the spherical indenter): the strain varies with the indentation depth in spherical indentation, which makes it possible to evaluate the indentation flow curve but very difficult to estimate the amount of pile-up/sink-in. Third, the plastic property is the work-hardening exponent, which indicates the extent of work-hardening behavior in plastic deformation.

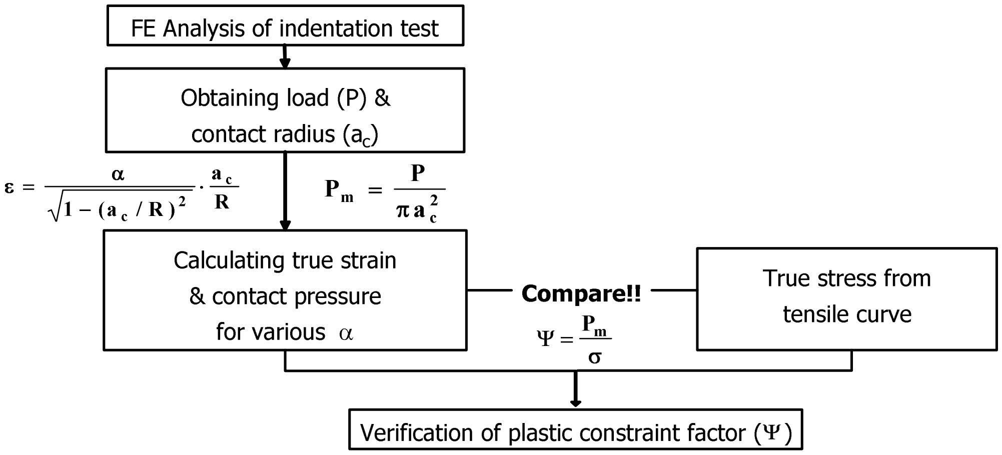
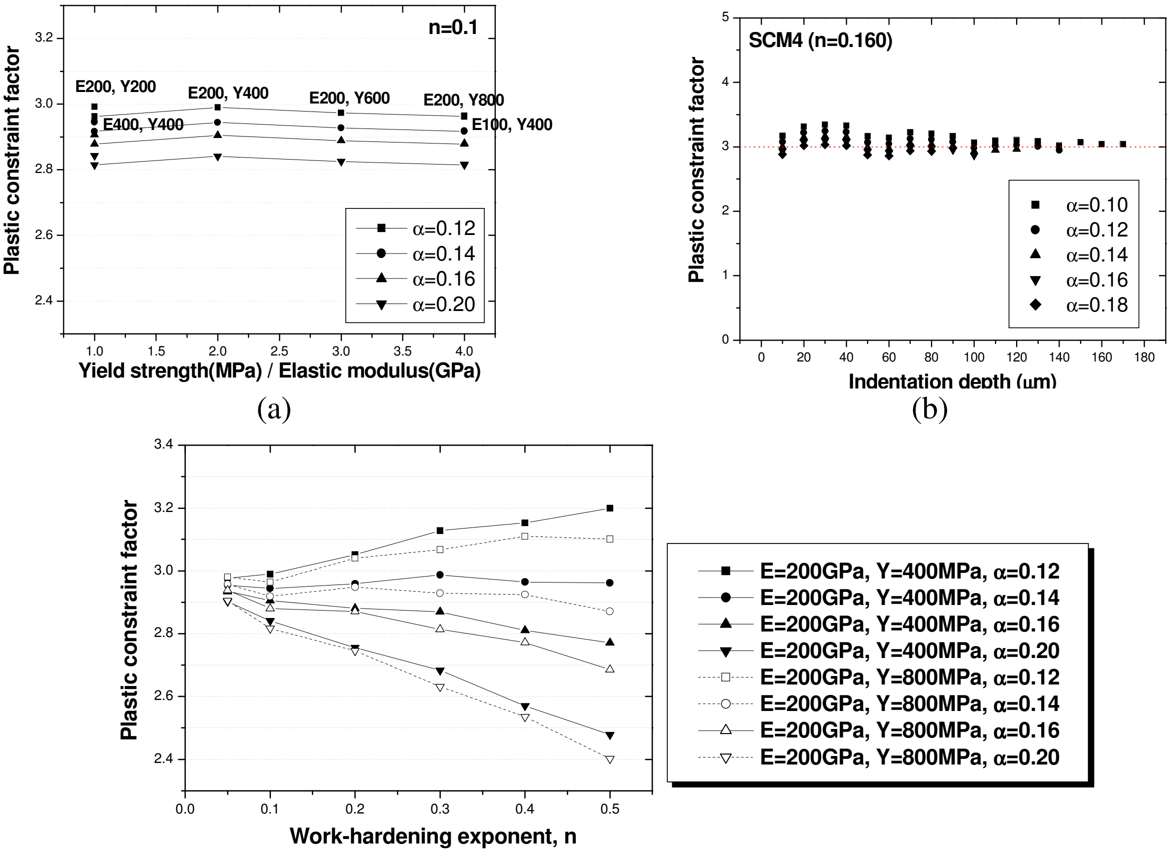


Fig. 4 Flow chart for verification of the proportional constants



(a) (b)

(c)

Fig. 5 The variation of α and ψ with
(a) elastic property, (b) strain level and (c) work-hardening exponent

As shown in Fig. 5 (a)-Fig. 5 (b), the plastic constraint factor does not vary with the elastic property or strain level. On the other hand, this factor shows a constant value about 3.0 only when the value of α is 0.14 in varying the work-hardening exponent. This tendency was observed for all simulated materials. The plastic constraint factor is known to be constant in the fully plastic deformation regime [5]. Our simulation was performed the load range of several tens of kgf. Generally, yielding occurs below 1gf with the spherical indenter of 1mm diameter used for the simulations. Therefore, all deformation occurs in the fully plastic regime. Due to the small elastic deformation, the elastic property does not affect the plastic constraint factor. Also, it has been proven that, in a metallic material, one of the plastic deformation phenomena, pile-up/sink-in, has no connection with the elastic property [6]. It is known that hardness (the same concept with the mean contact pressure) increases with indentation depth by similar stress increase characteristics [4]. This implies that the strain level does not affect the plastic constraint factor. However, the work-hardening exponent is a primary property determining a material's plastic deformation behavior, and by its nature the plastic constraint factor may be dependent on this exponent. Nevertheless, a constant value was observed at $\alpha = 0.14$. It is presumed that the representative strain is most appropriately calculated at $\alpha = 0.14$ because the plastic constraint factor 3.0 is consistent with the slip-line field theory [7, 8] and earlier research [1, 9]. The details and rationale for the mechanism will be the subjects of further study.

From the above results, we determined the optimum value of α and ψ as 0.14 and 3.0, respectively and derived the indentation flow curves. As shown in Fig. 6, the indentation flow curve and the uniaxial tensile curve overlap each other well. The same results were observed for all simulated real and imaginary materials, showing that our optimum values are valid for a large range of metallic materials.

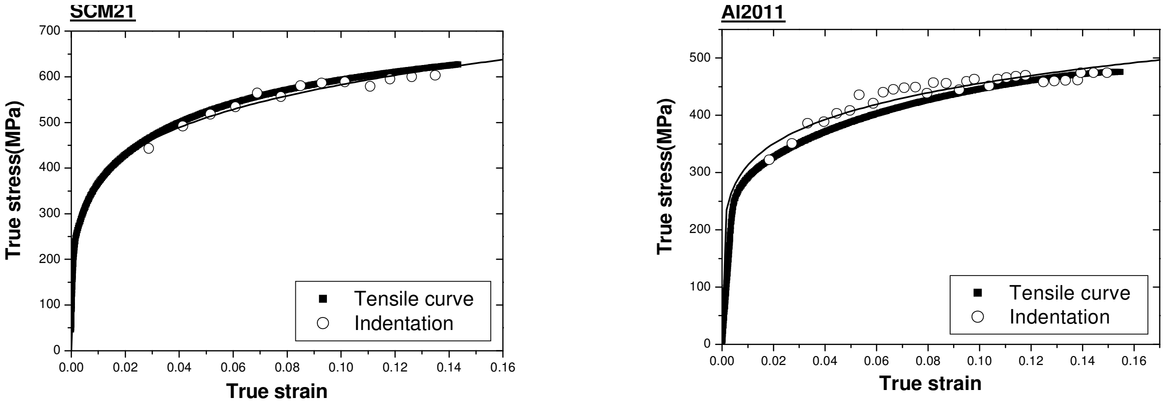


Fig. 6 Comparisons of the indentation flow curve and the uniaxial tensile curve of (a) SCM21 and (b) Al2011

Summary

The method of calculating the representative strain and representative stress for a spherical indentation was determined and verified from the physical concepts. The elastic property and the strain level do not affect the plastic constraint factor, which relates to plastic behavior and is a main parameter of the representative stress. This factor is a constant value of 3.0 when the proportional constant of the representative strain is 0.14 regardless of the work-hardening exponent, although the exponent is expected to control the factor. With these optimum values, the indentation flow curve and the uniaxial tensile curve overlap each other very well for 38 kinds of metallic materials.

Acknowledgement

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